1. Exercice 2 : Calculer $f'(a)$

1.1. $f : x \to 2x - 3 \quad ; \quad a = 0$.

$$f'(x) = 2 \qquad \text{donc} \qquad f'(0) = 2$$

($f'(x)$ est une fonction constante elle vaut 2 $\forall x$)

1.2. $f : x \to 3x^2 + 2x - 1 \quad ; \quad a = 2$.

$$f'(x) = 6x + 2 \qquad \text{donc} \qquad f'(2) = 14$$

1.3. $f : x \to \frac{x-2}{x-3} \quad ; \quad a = 2$.

$$f(x) = \frac{x-2}{x-3} = \frac{u}{v}$$
$$u = x - 2 \quad ; \quad u' = 1 \quad ; \quad v = x - 3 \quad ; \quad v' = 1$$
$$f'(x) = \frac{u'v - v'u}{v^2} = \frac{(x-3) - (x-2)}{(x-3)^2} = \frac{-1}{(x-3)^2}$$
$$f'(x) = \frac{-1}{(x-3)^2} \qquad \text{donc} \qquad f'(2) = -1$$

1.4. $f : x \to \sqrt{5-x} \quad ; \quad a = 4$.

$$f(x) = \sqrt{5-x} = \sqrt{u}$$
$$u = 5 - x \quad ; \quad u' = -1$$
$$f'(x) = \frac{u'}{2\sqrt{u}} = \frac{-1}{2\sqrt{5-x}} \qquad \text{donc} \qquad f'(4) = -\frac{1}{2}$$

2. Exercice 3 : Calculer $f'(x)$

2.1. $f : x \to (2-x)^3$.

$$f(x) = (2-x)^3 = u^3$$
$$u = 2 - x \quad ; \quad u' = -1$$
$$f'(x) = 3u^2u' = -3(2-x)^2$$

2.2. $f : x \to \frac{4}{x}$.

$$f(x) = \frac{4}{x} = 4 \times \frac{1}{x}$$
$$f'(x) = 4 \times \frac{-1}{x^2} = \frac{-4}{x^2}$$

2.3. $f : x \to \frac{-2}{x-1}$.

$$f(x) = \frac{-2}{x-1} = -2 \times \frac{1}{u}$$
$$u = x - 1 \quad ; \quad u' = 1$$
$$f'(x) = -2 \times \frac{-u'}{u^2} = -2 \times \frac{-1}{(x-1)^2} = \frac{2}{(x-1)^2}$$

2.4. $f : x \to \frac{2x-1}{x+2}$.

$$f(x) = \frac{2x-1}{x+2} = \frac{u}{v}$$
$$u = 2x - 1 \quad ; \quad u' = 2 \quad ; \quad v = x + 2 \quad ; \quad v' = 1$$
$$f'(x) = \frac{u'v - v'u}{v^2} = \frac{2(x+2) - (2x-1)}{(x+2)^2} = \frac{5}{(x+2)^2}$$

2.5. $f : x \to 3x - 5 + \frac{3}{2x}$.

$$f(x) = 3x - 5 + \frac{3}{2x} = 3x - 5 + \frac{3}{2} \times \frac{1}{x}$$
$$f'(x) = 3 + \frac{3}{2} \times \frac{-1}{x^2} = 3 + \frac{-3}{2x^2}$$

2.6. $f : x \to x^2 + \sqrt{x}$.

$$f(x) = x^2 + \sqrt{x}$$
$$f'(x) = 2x + \frac{1}{2\sqrt{x}}$$

2.7. $f : x \to \sqrt{5x-4}$.

$$f(x) = \sqrt{5x-4} = \sqrt{u}$$
$$u = 5x - 4 \quad ; \quad u' = 5$$
$$f'x) = \frac{u'}{2\sqrt{u}} = \frac{5}{2\sqrt{5x-4}}$$

3. Exercice 5 : Variation et dérivées de $f(x)$

3.1. $f : x \to -x^2 + 2x + 3$.

$$f(x) = -x^2 + 2x + 3$$
$$f'(x) = -2x + 2$$

x	$-\infty$		$+1$		$+\infty$
$f'(x)$		$+$	0	$-$	
$f(x)$		$\nearrow$	4	$\searrow$	

f'(x) = ax+b avec a = -2 < 0 donc négatif pour x < 1

3.2. $f : x \to \frac{x+3}{x-2}$.

$$f(x) = \frac{x+3}{x-2} = \frac{u}{v}$$
$$u = x + 3 \quad ; \quad u' = 1 \quad ; \quad v = x - 2 \quad ; \quad v' = 1$$
$$f'(x) = \frac{u'v - v'u}{v^2} = \frac{(x-2) - (x+3)}{(x-2)^2} = \frac{-5}{(x-2)^2}$$

x	$-\infty$		$+2$		$+\infty$
$f'(x)$		$-$	$\|\|$	$-$	
$f(x)$		$\searrow$	$\|\|$	$\searrow$	

3.3. $f : x \to \frac{x^2+x+1}{x+2}$.

$$f(x) = \frac{x^2+x+1}{x+2} = \frac{u}{v}$$

$$u = x^2+x+1 \quad ; \quad u' = 2x+1 \quad ; \quad v = x+2 \quad ; \quad v' = 1$$

$$f'(x) = \frac{u'v - v'u}{v^2} = \frac{(2x+1)(x+2) - (x^2+x+1)}{(x+2)^2} = \frac{2x^2+4x+x+2-x^2-x-1}{(x+2)^2} = \frac{x^2+4x+1}{(x+2)^2}$$

$$\text{racines de } x^2+4x+1 : -2 \pm \sqrt{3}$$

$$f'(x) = \frac{(x+2-\sqrt{3})(x+2+\sqrt{3})}{(x+2)^2}$$

x	$-\infty$		$-2-\sqrt{3}$		$-2+\sqrt{3}$		$+\infty$
$x+2-\sqrt{3}$		$-$		$-$	0	$+$	
$x+2+\sqrt{3}$		$-$	0	$+$		$+$	
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$		$\nearrow$		$\searrow$		$\nearrow$	